Exercises for Midterm exam - Magnetism

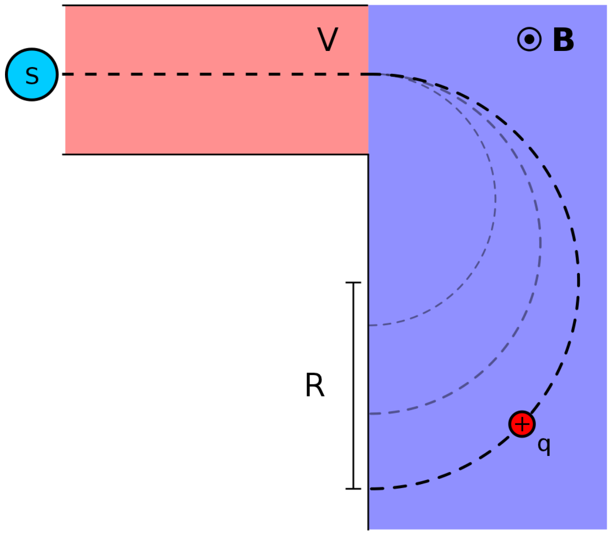

Figure 1

Exercise 0.1: Mass Spectrograph

A mass spectrograph is used to measure the masses of ions, or to separate ions of different masses. In one design for such an instrument, ions with mass m and charge q are accelerated trough a potential difference V. They then enter a uniform magnetic field that is perpendicular to their velocity, and they are deflected in a semicicular path of radius R. A detector measures where the ions complete the semicircle and from this it is easy to cacluate R. The situation where a source S is emitting ions is shown in figure 1. In the red region there is a potential difference, while in the blue region there is a uniform magnetic field.

a) Derive the equation for calculating the mass of the ion from measurements of B, V, R and q.

Solution:

The speed of the ions when they enter the region of a uniform magnetic field is found by energy conservation;

$$qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV}{m}}. \tag{1}$$

When the ion enter this region it will feel a magnetic force

$$\mathbf{F}_b = q\mathbf{v} \times \mathbf{B}. \tag{2}$$

Now this force is always perpendicular to $\mathbf{v}$, so the acceleration is always perpendicular to $\mathbf{v}$. This is a characteristic of circular motion, so that the ion will start going in a circle at once it enters the region of uniform $\mathbf{B}$. Furthermore $\mathbf{B}$ is always perpendicular to $\mathbf{v}$ so the magnitude is

$$F_b = qvB = ma = m\frac{v^2}{R}. \tag{3}$$

Thus

$$qBR = mv = m\sqrt{\frac{2qV}{m}} = \sqrt{2qVm} \tag{4}$$

or equivalently

$$m = \frac{qB^2R^2}{2V}. \tag{5}$$

Answer:

$$m = \frac{qB^2R^2}{2V}. \tag{6}$$

b) What potential difference V is needed so that singly ionized ^{12}C atoms will have $R = 50.0\,\text{cm}$ in a $0.150\,\text{T}$ magnetic field?

Solution:

Rearranging the formula found in (b) we find

$$V = \frac{qB^2R^2}{2m} \tag{7}$$

The mass of Carbon-12 is $12\,\text{u}$ and if it's singly ionized it obtains a charge e. Thus the necessary potetial is

$$V = \frac{1.6\times10^{-19}\times(0.150)^2\times\left(50.0\times10^{-2}\right)}{2\times12\times1.66\times10^{-27}}\,\text{V} = 22.5904\,\text{kV}. \tag{8}$$

Answer:

$$V = 22.5904\,\text{kV}. \tag{9}$$

c) Suppose the beam consists of a mixture of ^{12}C and ^{14}C ions. If V and B have the same values as in part (b), calculate the separation of these two isotopes at the detector. Do you think that this beam separation is sufficient for the two ions to be distinguished?

Solution:

Carbon-14 has a mass of 14 u. Again rearranging the formula in (a) and (b) differences in radii of the two semicircles will be

$$\Delta R = \sqrt{\frac{2m_{^{14}\mathrm{C}}V}{qB^2}} - \sqrt{\frac{2m_{^{12}\mathrm{C}}V}{qB^2}} = 4\,\mathrm{cm}. \tag{10}$$

This beam separation should allow for easy separation of the two ions.

Answer:

$$\Delta R = 4\,\mathrm{cm} \tag{11}$$

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Exercise 0.2: Force on a Semicircular Wire

Figure 2 shows a conducting wire which enters a region of a uniform magnetic field **B** pointing out of the page. In this region the wire takes the form of a semicirle. The direction of the current running trough it is shown by the arrows. Find the net force on the conducting wire.

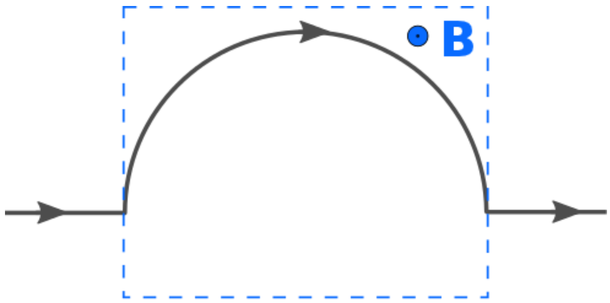

Figure 2

Solution:

The wire only feels a force in the region where there is a magnetic field. An infinitesimal segment of wire feels the force $d\mathbf{F} = Id\mathbf{l} \times \mathbf{B}$. Now as shown in figure 3 the contribution to the y-component of this force is given by

$$dF_y = |d\mathbf{F}| \sin\theta \tag{12}$$

where $|d\mathbf{F}| = IBdl = IBRd\theta$ so that the magnitude of the entire y-component of force is given by

$$F_y = \int dF_y = IBR \int_0^{\pi} \sin\theta d\theta = 2IBR. \tag{13}$$

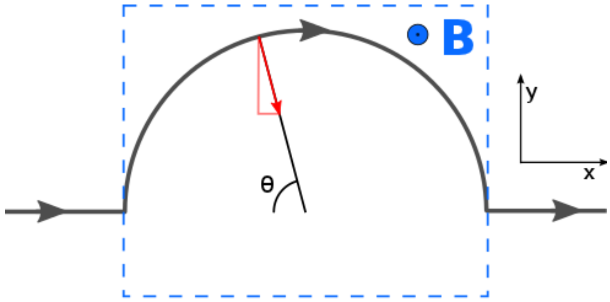

Figure 3

The x-component is given by

$$F_x = IBR \int_0^{\pi} \cos\theta d\theta = 0 \tag{14}$$

which we also could have seen from the symmetry the problem. Furhermore by the right hand rule this force must point in the negative y-direction. Therefore we conclude that

$$\mathbf{F} = -2IBR\hat{\mathbf{j}}. \tag{15}$$

Answer:

$$\mathbf{F} = -2IBR\hat{\mathbf{j}}. \tag{16}$$