Problems from D. J. Griffiths' *Introducion to electrodynamics*, 3rd edition

Example 1.1

Let $\mathbf{C} = \mathbf{A} - \mathbf{B}$ and calculate the dot product of $\mathbf{C}$ with itself. Relate your answer to the law of cosines.

Problem 1.2

Is the cross product associative, i.e. $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} \stackrel{?}{=} \mathbf{A} \times (\mathbf{B} \times \mathbf{C})$? If so, *prove* it; if not, provide a counterexample.

Example 1.2

Find the angle between the face diagonals of a cube.

Example 1.3

Find the gradient of $r = \sqrt{x^2 + y^2 + z^2}$ (the magnitude of the position vector).

Problem 1.11

Find the gradients of the following functions:
a) $f(x, y, z) = x^2 + y^3 + z^4$.
b) $f(x, y, z) = x^2 y^3 z^4$.
c) $f(x, y, z) = e^x \sin(y) \ln(z)$.

Problem 1.13

Let $\mathbf{r}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and let r be its length. Show that
a) $\nabla(r^2) = 2\mathbf{r}$.
b) $\nabla(1/r) = -\hat{\mathbf{r}}/r^2$.
c) What is the *general* formula for $\nabla(r^n)$?

Problem 1.15

Calculate the divergence of the following vector functions:
a) $\mathbf{v}_a = x^2\hat{\mathbf{x}} + 3xz^2\hat{\mathbf{y}} - 2xz\hat{\mathbf{z}}$.
b) $\mathbf{v}_b = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3zx\hat{\mathbf{z}}$.
c) $\mathbf{v}_c = y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}$.

Problem 1.18

Calculate the curls of the vector functions in Problem 1.15.

Problem 1.19

Construct a vector function that has zero divergence and zero curl everywhere. (A *constant* will do the job, of course, but make it something a little more interesting than that!)

Problem 1.28

Calculate the line integral of the function $\mathbf{v} = x^2\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + y^2\hat{\mathbf{z}}$ from the origin to the point $(1, 1, 1)$ by three different routes:

a) $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 0) \to (1, 1, 1)$;
b) $(0, 0, 0) \to (0, 0, 1) \to (0, 1, 1) \to (1, 1, 1)$;
c) The direct straight line.
d) What is the line integral around the closed loop that goes *out* along path a) and *back* along path b)?

Problem 1.30

Calculate the volume integral of the function $T = z^2$ over the tetrahedron with corners at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

Problem 1.31

Check the fundamental theorem for gradients, using $T = x^2+4xy+2yz^3$, the points $\mathbf{a} = (0, 0, 0)$, $\mathbf{b} = (1, 1, 1)$ and the following three paths:

a) $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 0) \to (1, 1, 1)$;
b) $(0, 0, 0) \to (0, 0, 1) \to (0, 1, 1) \to (1, 1, 1)$;
c) the parabolic path $z = x^2$; $y = x$.

Example 1.10

Check the divergence theorem using the function $\mathbf{v} = y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}$ and the unit cube situated at the origin.

Problem 1.32

Test the divergence theorem for the function $\mathbf{v} = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3zx\hat{\mathbf{z}}$. Take as your volume the cube with side lengths 2, one corner at the origin and all of the cube in the first octant.

Problem 1.33

Check the fundamental theorem of curls for the function $\mathbf{v} = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3zx\hat{\mathbf{z}}$, using the triangle defined by the corners $(0, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.

Problem 1.49

a) Let $\mathbf{F}_1 = x^2\hat{\mathbf{z}}$ and $\mathbf{F}_2 = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$. Calculate the divergence and curl of $\mathbf{F}_1$ and $\mathbf{F}_2$. Which one can be written as the gradient of a scalar? Find a scalar potential that does the job. Which one can be written as the curl of a vector? Find a suitable vector potential.

b) Show that $\mathbf{F}_3 = yz\hat{\mathbf{x}} + zx\hat{\mathbf{y}} + xy\hat{\mathbf{z}}$ can be written both as the gradient of a scalar and as the curl of a vector. Find scalar and vector potentials for this function.

Problem 1.53

Check the divergence theorem for the function $\mathbf{v} = r^2\cos\theta\,\hat{\mathbf{r}} + r^2\cos\phi\,\hat{\boldsymbol{\theta}} - r^2\cos\theta\sin\phi\,\hat{\boldsymbol{\phi}}$, using as your volume one octant of the sphere of radius R. Make sure you include the *entire* surface. [*Answer*: $\pi R^4/4$]

Problem 1.55

Compute the line integral of $\mathbf{v} = 6\hat{\mathbf{x}} + yz^2\hat{\mathbf{y}} + (3y+z)\hat{\mathbf{z}}$ along the triangular path $(0,0,0) \to (0,1,0) \to (0,0,2) \to (0,0,0)$. Check your answer using the fundamental theorem of curls. [*Answer*: $8/3$]