On the concept of Dipole Moment

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The concept of the *dipole moment* are often introduced only in the case of two point charges separated by a distance d and it often not emphasized of what use this concept has when we're dealing with other charge distributions. We then defined the dipole moment as

$$\mathbf{p} = q\mathbf{d},$$

where $\mathbf{d}$ is the separation vector between the charges.

If the concept applied only to two point charges it would not be of much use since most molecules and atoms, for which the idea of *polarization* becomes important, are not only two point charges separated by a distance. Luckily the fact is that is is an important and much more general idea and I think this should be emphasized in more textbooks.

We'll first need a usefull results about torques. Let's say that I wanted to take the torque about my origin, while you wanted to take it about yours. Then I would compute

$$\boldsymbol{\tau} = \sum_i \mathbf{r_i} \times \mathbf{F_i}$$

and you would compute

$$\boldsymbol{\tau}' = \sum_i \mathbf{r'_i} \times \mathbf{F_i}$$

now your origin is related to mine by some separation distance $\mathbf{a}$, so $\mathbf{r_i} = \mathbf{a} + \mathbf{r'_i}$ which means that your torque will be

$$\boldsymbol{\tau}' = \sum_i (\mathbf{r_i} - \mathbf{a}) \times \mathbf{F_i} = \sum_i \mathbf{r_i} - \mathbf{a} \sum_i \mathbf{F_i} = \boldsymbol{\tau} - \mathbf{a} \sum_i \mathbf{F_i}.$$

So if the sum of all forces are zero we will compute the same torque. An equivalent way to say this is that *if the forces sum to zero the torque is the same taken about any point in space.*

Now back to dipole moments. What we found when studying the two opposite charges separated by a distance d was that we could neatly write $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$ and $U(\theta) = -\mathbf{p} \cdot \mathbf{E}$. These equations tell us how such an arrangement responds when placed in a uniform electric field[1] and it's this together with the theory of the dipole potential that make the concept of the dipole moment important. In words they tell us that when such an arrangement is placed in an electric field, then it will tend to rotate and align $\mathbf{p}$ parallel with $\mathbf{E}$. And the bigger $|\mathbf{p}|$ is, the more tendency the arrangement has to rotate and the more energy is stored. So are they also valid for other configurations? The answer is yes. If we define the dipole moment to be

[1] Since the separation between charges is usually really small, then this will be a good approximation also for non-uniform fields as long as the field does not vary rapidly in space

$$\mathbf{p} = \sum_i \mathbf{r_i} q_i$$

and if we require that $\sum_i q_i = 0$ then all these equations hold and the dipole moment, just like torque when $\sum_i \mathbf{F_i} = 0$, is a quantity which is independent of which point you choose as your origin. Let's prove the second claim first. Like before I compute $\mathbf{p}$ while you compute $\mathbf{p}'$ and our origins are separated by $\mathbf{a}$. We then have

$$\mathbf{p}' = \sum_i \mathbf{r_i}' q_i = \sum_i (\mathbf{r_i} - \mathbf{a}) q_i = \sum_i \mathbf{r_i} q_i - \mathbf{a} \sum_i q_i = \mathbf{p} - \mathbf{a} \sum_i q_i.$$

So if the net charge is zero $\mathbf{p}' = \mathbf{p}$. Now, something that has net charge zero in a uniform field will also have a net force of zero. Therefore we can take the torque about any point and we find

$$\boldsymbol{\tau} = \sum_i \mathbf{r_i} \times \mathbf{F_i} = \sum_i \mathbf{r_i} \times q_i \mathbf{E} = \left(\sum_i \mathbf{r_i} q_i \right) \times \mathbf{E} = \mathbf{p} \times \mathbf{E}.$$

The energy equation is derived from considering the integral $\int_0^\theta \tau d\theta$ so it's clear that this will also hold in the general case.

The conclusion then is that dipole moment is a very usefull concept especially in the case when the net charge is zero and it becomes important in the study of matter since this is one of the defining properties of *atoms* and *molecules*.